WATERSCAPE INTERNATIONAL GROUP

Energy and Resources Equations
Fact Sheet #001-01

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Waterscape International Group is a 501(c)3 tax-exempt organization founded in California for research and educational purposes.

General Prefixes

10 deka(da)	10^{-1} deci(di)
10^2 hecto(h)	10^{-2} centi(c)
10^3 kilo(k)	10^{-3} milli(m)
10^6 mega(M)	10^{-6} micro(μ)
10^9 giga(G)	10^{-9} nano(n)
10^{12} tera(T)	10^{-12} pico(p)
10^{15} peta(P)	10^{-15} femto(f)
10^{18} exa(E)	10^{-18} atto(a)

Conversions

1 year = 3.1536E7 seconds
1 joule = 1 kg m^2/sec^2 = 1 newton-meter
1 joule = 1 watt-second (Ws)
3.6E6 joules = 1 kWh
1 pascal = 1 N/m^2 = 1 J/m^3
1 newton (N) = 1 kg m/sec^2
Stefan-Boltzmann (σ) = 5.669E-8 $J/(m^2\text{-}K^4\text{-sec})$
A = 6.02E23 molecules/mole
Ideal Gas (R) = 8.310 J/mole-K
Speed of light (c) = 2.9979E8 m/sec
Gravitation (G) = 6.67E-11 $N\text{-}m^2/kg^2$
Acceleration = 9.8 m/sec^2

Earth

Mass Earth = 5.98E24 kg
Mass Atm = 5.14E18 kg
Mass Strat = 0.5E18 kg
Mass Oceans = 1.4E21 kg
Mass water in Atm = 1.3E16 kg
Mass surface fresh water = 1.26E17
Mass living org (dry) = 1.3E15
Moles dry air in atm = 1.8E20
p(atm) = ½ @ 5600 m
Top trop = 12,000 m
Mean ocean depth = 3,730 m
Mixed layer = 75 m
Cont. Elev. = 840 m
Earth Area = 5.10E14 m^2
Cont. = 1.48E14 m^2
Ocean Area = 3.61E14 m^2
Ocean Volume = 1.35E18 m^3
Ocean mixed-layer = 2.7E16 m^3
Earth Density = 5500 kg/m^3
Surface seawater density = 1,026 kg/m^3
Mean surface air temp = 288 K

Astronomical

Earth⇔Sun = 1.495E11 m
Earth Radius = 6.38E6, polar 6.36
Sun Radius = 6.96E8 m
Sun Mass = 1.99E30 kg
Earth⇔Moon = 3.84E8
Moon Radius = 1.74E6 m
Moon Mass = 7.34E22 kg
Lunar revolution = 2.36E6 sec
Nucleons in the universe = 10^{80}
Radius of universe = 10^{26} m

Air

28.96 g/mole; 22.4 l/mole
specific heat = 1,004.2 J/kg ºC
Density = 1.293 kg/m^3
Conductivity = 0.0209 W/m ºC

Molecule	Moles	Mass
N_2	0.7808	0.7549
O_2	0.2095	0.2314
Ar	0.0093	0.0128
CO_2	370 ppm	516 ppm
He	5.2 ppm	0.7 ppm
H_2	0.5 ppm	0.03 ppm
N_2O	0.3 ppm	0.45 ppm
O_3	0.01 ppm	0.015 ppm
NO_2	0.2 ppb	0.3 ppb
SO_2	0.2 ppb	0.4 ppb
H_2S	0.05 ppb	0.05 ppb
NO	0.05 ppb	0.05 ppb
NH_3	0.05 ppb	0.03 ppb

Water

0	999.87 kg/m^3
3.98	1,000 kg/m^3
25	997.07 kg/m^3

Latent heat Fusion @ 0 = 3.33E5 J/kg; 79.6 cal/g
Latent heat vapor. @100= 2.258E6 J/kg; 539.6 cal/g
@ 17 = 2.459E6 J/kg
Specific heat of liquid water @ 15 = 4,184 J/kg ºC
Specific heat water vapor @ 100 = 2,008.3 J/kg ºC

Coefficient heat conductivity @ 17 = 0.595 W/m ºC

Stocks of Water (10^{15} m^3)

Oceans	1,350
Ice	29
Groundwater	8.3
Fresh lakes	0.125
Saline lakes	0.104
Soil	0.067
Atmosphere	0.013
Biomass	0.003
Rivers	0.001

Flows of water (10^{12} m^3/yr)

World precip. Land	108	
In sea		410
Et land		62
Et sea		456
Runoff	46	

Energy (10^{12} W)

Sun radiates	3.7E14
Solar radiation top atm	175,000(343)
Reflected back from earth	53,000
Reflected back from atm	46,000
Solar radiation in atm	44,000
Latent heat earth→atm	42,000
IR earth→ space	10,200
Convection surf → atm	8,600
Ocean currents	~1250
NPP	100
Geo→ earth surface	30
World energy con	10(2.5)
Food	0.55
Electricity	0.87

(1E6 J/kg)

Dry biomass	16
Wood	15
Fat	38
Gas	48
Oil	43
Coal	29.3

Reaction	10^{-pK}
$H_2O \Leftrightarrow H^+ + OH^-$	10^{-14}
$H_2CO_3 \Leftrightarrow H^+ + HCO_3^-$	$10^{-6.35}$
$HCO_3^- \Leftrightarrow H^+ + CO_3^{-2}$	$10^{-10.33}$
$HCl \Leftrightarrow H^+ + Cl^-$	$10^{3.0}$
$H_2SO_4 \Leftrightarrow H^+ + HSO_4^-$	$10^{3.0}$
$HSO_3^- \Leftrightarrow H^+ + SO_3^{-2}$	$10^{-1.9}$
$HNO_3 \Leftrightarrow H^+ + NO_3^{-1}$	$10^{3.0}$
$H_2SO_3 \Leftrightarrow H^+ + HSO_3^-$	$10^{-1.77}$
$HSO_3^- \Leftrightarrow H^+ + SO_3^{-2}$	$10^{-7.21}$
$NH_3 + H_2O \Leftrightarrow NH_4^+ + OH^-$	$10^{-4.74}$
$H_3BO_3 \Leftrightarrow H^+ + H_2BO_3^-$	$10^{-9.3}$

Equilibrium ratio	K_H (moles/liter-atm)
$[H_2SO_3]/p(SO_2)$	$10^{0.096}$
$[H_2CO_3]/p(CO_2)$	$10^{-1.47}$
$[HNO_3]/p(NO_2)$	$10^{-1.6}$
$[NH_3]/p(NH_3)$	$10^{1.76}$
$[CO]/p(CO)$	$10^{-3.0}$
$[N_2O]/p(N_2O)$	$10^{-1.59}$
$[H_2S]/p(H_2S)$	$10^{-0.97}$

Solid	solubility product ($moles^2/liter^2$)
Calcite	$[Ca^{+2}][CO_3^{-2}] = 10^{-8.42}$
Aragonite	$[Ca^{+2}][CO_3^{-2}] = 10^{-8.22}$
	$[Ca^{+2}][CO_3^{-2}] = 10^{-6.05}$ seawater
gypsum	$[Ca^{+2}][SO_4^{-2}] = 10^{-4.6}$
dolomite	$[Ca^{+2}][Mg^{+2}][CO_3^{-2}]^2 = 10^{-16.7}$

Reaction	Constant (liters/mole)
$2H^+ + CuO \Leftrightarrow Cu^{+2} + H_2O$	$10^{7.7}$
$3H^+ + Al(OH)_3 \Leftrightarrow Al^{+3} + 3H_2O$	$10^{8.5}$

Biomass	Living	Dead	NPP
	(10^{12} kg (C))		(10^{12} kg (C)/yr)
Cont.	560	1500	50
Marine	2	2,000	25

Wood C:N Ratio 200:1
Biomass C:N Ratio 10:1
Biomass $H_{20}O_{10}C_{10}N$

Ecosystem type	area 10^{12} m^2	biomass kg(c)/m^2	NPP per year
Tropical forests	24.5	18.8	0.83
Temperate forests	12	14.6	0.56
Boreal forests	12	9	0.36
Woodland	8	2.7	0.27
Savanna	15	1.8	0.32
Grassland	9	0.7	0.23
Tundra	8	0.3	0.065
Desert	18	0.3	0.032
Rock, ice	24	0.01	0.015
Cultivated land	14	0.5	0.29
Swamp	2	6.8	1.13
Lake and stream	2.5	0.01	0.23
Ocean	332	0.0014	0.057
Upwelling zones	0.4	0.01	0.23
Cont. shelf	26.2	0.005	0.16
Algal bed and reef	0.6	0.9	0.9
Estuaries	1.4	0.45	0.81

Resulting formula:
$\sigma T_s^4 = 3\Omega(1\text{-}a)/4 - [F_c + 1.5F_e + 1.7F_s + 2F_w]$
F_c = 17 w/m^2 (convective heat flow)
F_e = 80 w/m^2 (latent heat)
F_s = 86 w/m^2 (absorbed in atm)
F_w = 20 w/m^2 (radiated to space)
T_s = temperature of the surface
T_o = first, lower layer
T_1 = second, higher layer
The 2 is the two-layer system
The 3 is n layers + 1
Empirically we know that this energy gets dumped in the lower troposphere. Most of the water is in the lower atmosphere.
$\Omega/4$ = 343 w/m^2
a = 0.3
p166

$W + \Omega/4 = a(\Omega/4) + \sigma T_o^4 + F_w$
$2\sigma T_o^4 = \sigma T_1^4 + 0.5F_e + 0.7F_s$
$2\sigma T_1^4 = \sigma T_o^4 + \sigma T_s^4 - F_w + F_c + 0.5F_e + 0.3F_s + W$

$\sigma T_o^4 = 220.1$ W/m^2; T_o = 249.6 K
$\sigma T_1^4 = 340$ W/m^2; T_1 = 278.3 K
$\sigma T_s^4 = 397.1$ W/m^2; T_s = 289.3 K

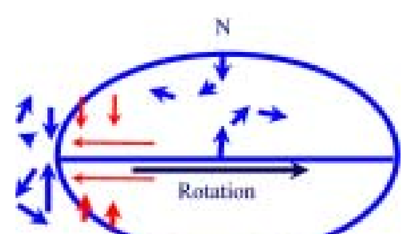
Fin (p) = pollutant flow, mass/time
Fin (water) = water flow, mass/time
Concentration in lake is Mp/Mwater
dM(water)/dt = 0; Fin (water) = Fout (water)
$\tau_{water} = M_{water}/F_{in,water}$
$\tau_{(p)} = Mp/F_{out,p}$ (only applies when Fin, p = Fout, p)
Proceed, assuming Steady-state
Fout, p = Mp/Mw*(Fout, w)
In other words, Fout, p = concentration * total outflow
τ_p = Mp/[Mp/Mw * (Fout,w)] = Mw/Fout, w = τ_{water} (only applies at steady state.
Additionally, if Et applied, τ_p would be > τ_{water}

General equation

dMp/dt = Fin, p – Mp/Mw * (Fout, w)
Of the general form, dx/dt = a –bx; a linear, donor-controlled equation.

$$\int \frac{dx}{a - bx} = \int dt$$

$Mp(t) = Mp(0) + [\tau_{water} * F_{in,p} - Mp(0)][1 - e^{-t/\tau water}]$
t→0, Mp(0) = Mp(0) + [τ_{water} * Fp,in –Mp(0)]*[0]
t→∞, Mp(∞) = Mp(0) + [τ_{water} * Fp,in –Mp(0)]*[1]
Mp(∞) = τ_{water} * Fp,in (this only applies when there is no evaporative loss of water and $\tau_p = \tau_{water}$)
In general at SS, Mp(∞) = $\tau_p * F_{p,in}$
Another approach for steady state situations,
dMp/dt = Fin, p – Mp/τ_{water}
0 = Fin, p – Mp/τ_{water}
Mp = Fin, p * τ_{water}, essentially applies because we know $\tau_p = \tau_{water}$



Coriolis Effect

1	2 IIA 2A	3 IIIB 3B	4 IVB 4B	5 VB 5B	6 VIB 6B	7 VIIB 7B	8	9	10	11 IB 1B	12 IIB 2B	13 IIIA 3A	14 IVA 4A	15 VA 5A	16 VIA 6A	17 VIIA 7A	
1 H 1.008																	2 He 4.003
3 Li 6.941	4 Be 9.012											5 B 10.81	6 C 12.01	7 N 14.01	8 O 16.00	9 F 19.00	10 Ne 20.18
11 Na 22.99	12 Mg 24.31											13 Al 26.98	14 Si 28.09	15 P 30.97	16 S 32.07	17 Cl 35.45	18 Ar 39.95
19 K 39.10	20 Ca 40.08	21 Sc 44.96	22 Ti 47.88	23 V 50.94	24 Cr 52.00	25 Mn 54.94	26 Fe 55.85	27 Co 58.47	28 Ni 58.69	29 Cu 63.55	30 Zn 65.39	31 Ga 69.72	32 Ge 72.59	33 As 74.92	34 Se 78.96	35 Br 79.90	36 Kr 83.80

Example of Carbonate System

Assumes unlimited supply of $CaCO_3$

$p(CO_2) = 370$ ppm(v)

$[H_2CO_3] = p(CO_2)10^{-1.47}$ @ $p(CO_2)$ of 370, $[H_2CO_3] = 10^{-4.90}$

$[H+][HCO_3^-] = 10^{-6.35}[H_2CO_3]$ $[HCO_3^-] = (10^{-6.35}[H_2CO_3])/[H+]$ $[HCO_3^-] = 10^{-11.25}/[H+]$

$[H+][CO_3^{2-}] = 10^{-10.33}[HCO_3^-]$ $[CO_3^-] = (10^{-10.33}[HCO_3^-])/[H+]$ $[CO_3^-] = 10^{-21.6}/[H+]^2$

$[H+][OH^-] = 10^{-14}$ $[OH^-] = 10^{-14}/[H+]$ $[OH^-] = 10^{-14}/[H+]$

$[Ca^{+2}][CO_3^{-2}] = 10^{-8.42}$ $[Ca^{+2}] = 10^{-8.42}/[CO_3^{-2}]$ $[Ca^{+2}] = 10^{-8.42}/(10^{-21.6}/[H+]^2)$ $[Ca^{+2}] = 10^{13.18}[H+]^2$

Full Carbonate System equation

$[H+] + 2[Ca^{+2}] = [OH^-] + 2[CO_3^-] + [HCO_3^-]$

$[H+] + 2(10^{13.18}/[H+]^2) = 10^{-14}/[H+] + 2(10^{-21.6}/[H+]^2) + 10^{-11.25}/[H+]$

Remember $[HCO_3^-] \sim [H_2CO_3]$ at a pH of 6.35

$[CO_3^{2-}] \sim [HCO_3^-]$ at a pH of 10.33

Trick for Acid Dissociation

Add 0.1 g H_2SO_4 to 1 L water $pK=-\log_{10}K$, $K=10^{-pK}$

0.001 M

$$\frac{[H^+][HSO_4^-]}{[H_2SO_4]} = K_1 = 10^3$$

$$\frac{[H^+][SO_4^{-2}]}{[HSO_4^-]} = K_2 = 10^{-1.9}$$

If $-\log_{10}$[acid] > pK of that reaction, then full dissociation.

1. $-\log[0.001] > -\log[10^3]$
2. 3 > -3 fully dissociates and goes into second
3. $-\log[0.001] > -\log[10^{1.7}]$
4. 3 > 1.7, also fully dissociates
5. $pH = -\log_{10}[H^+] = 0.002 \rightarrow pH = 2.7$

$ALK = [HCO_3^-] + 2[CO_3^{-2}] + [OH^-] - [H^+]$ OR

$2[Ca^{+2}] + 2[Mg^{+2}] + [Na^+] + [K^+] - 2[SO_4^-] - [NO_3^-]$

for pH range of 6-8, $[ALK] \approx [HCO_3^-]$

$$[ALK] = \frac{K_1 K_H * p(CO2)}{[H^+]} - [H^+]$$

Acid Systems -- HNO_3

What is the pH?

0.63 g of HNO_3 added to 1 liter of water

= 0.01 moles/liter

$$\frac{[H^+][NO_3^-]}{[HNO_3]} = K_{HNO3} = 10^1$$

$$[HNO_3]*10 = [H^+][NO_3^-]$$

$$[0.01]*10 = [H^+][NO_3^-]$$

$$[H^+] = \frac{10^{-14}}{[H^+]} + \frac{0.01}{1 + \frac{[H^+]}{10}}$$